

Solutions

1.8: Operations on Functions

Basic Operations: Given two functions $f(x)$ and $g(x)$ you can create new functions using the basic operations on real numbers. $(+, -, \cdot, \div)$

Examples: Suppose $f(x) = 5x - 1$, $g(x) = 3x + 2$ and $h(x) = x^2 + 8$. Find the following functions.

- $f + g(x) = (5x - 1) + (3x + 2) = 8x + 1$
- $h - f(x) = (x^2 + 8) - (5x - 1) = x^2 - 5x + 9$
- $f \cdot g(x) = (5x - 1)(3x + 2) = 15x^2 + 7x - 2$

Composition of Functions: Given two function $f(x)$ and $g(x)$ you can create a new function by putting the output of one function as the input of the second function. This is the idea behind composition of functions.

Definition: For two function $f(x)$ and $g(x)$, the composite function $f \circ g$, also called the composition of f with g is defined by

$$f \circ g(x) = f(g(x)).$$

Example 1: If $f(t) = t^2$ and $g(t) = t + 2$, find

- (a) $f(t+1) = (t+1)^2 = t^2 + 2t + 1$
- (b) $f(t+h) = (t+h)^2 = t^2 + 2th + h^2$
- (c) $f(g(t)) = (t+2)^2 = t^2 + 4t + 4$
- (d) $g(f(t)) = t^2 + 2$

Example 2: If $f(x) = e^x$ and $g(x) = 5x + 1$, find

(a) $f \circ g(x) = e^{5x+1}$

(b) $g \circ f(x)$

u
 $5e^x + 1$

Example 3: Using the following table, find $f \circ g(0)$, $g \circ f(0)$, $f(g(1))$, $f(f(0))$, and $g(f(1))$.

x	0	1	2	3
$f(x)$	3	1	-1	-3
$g(x)$	0	2	4	6

$$f \circ g(0) = f(0) = 3$$

$$g \circ f(0) = g(3) = 6$$

$$f(g(1)) = f(2) = -1$$

$$f(f(0)) = f(3) = -3$$

$$g(f(1)) = g(1) = 2$$

Example 4: Use a new variable u for the inside functions to express each of the following as a composite function:

(a) $y = \ln(3t)$

(b) $w = 5(2x + 3)^2$

(c) $P = e^{-0.03t}$

(a) $y = \ln(u)$ where $u = 3t$

(b) $w = 5u^2$ where $u = 2x + 3$

(c) $P = e^u$ where $u = -0.03t$